Questions discussed on 20/9

1. If f is holomorphic on G, and f is injective, then $f' \neq 0$.
2. Are there any holomorphic function on $\mathbb{C} \setminus \{0\}$ such that $|f(z)| \geq \frac{1}{\sqrt{|z|}}$?
3. Suppose f is nonconstant entire function such that $f(0) = 0$ and $\{z : |f(z)| < M\}$ is connected for any $M > 0$. Show that there exists $c, n \in \mathbb{N}$ such that $f(z) = cz^n$.
4. Suppose f is entire function such that $Re(f) \geq C$ for some $C \in \mathbb{R}$, show that f is constant.

Extra question: Show that there are no holomorphic function on $\mathbb{C} \setminus \overline{\mathbb{D}}$ such that

$$|f(z)| \geq e^{|z|} \quad \forall |z| \geq 1.$$